Dynamical dark energy in cosmological holography

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Outline

Observations of accelerated expansion

Information and temperature in holography

New gravitational physics from the Hubble horizon

Confrontation with data: dark energy and dark matter

Conclusions
Expanding Universe

![Graph showing the relationship between velocity and distance](image)

**Figure 1**

*Velocity- Distance Relation among Extra-Galactic Nebulae.*

Hubble, E., 1929, PNAS, 168, 73

\[ v = HD \]
Distances from expansion velocities


$z \approx 0.1$

$D \approx 400$ Mpc
The universe is shaped by the mean density relative to the critical density.
3-flat Friedmann-Lemaître-Robertson-Walker (FLRW) line-element

\[ ds^2 = -dt^2 + a^2(t) \left( dx^2 + dy^2 + dz^2 \right) \]

\[ H = \frac{\dot{a}}{a}, \quad c = H R_H \]

\[ R_H = \frac{c}{H} \cong 4.3 \text{ Gpc} \]

An essentially 3-flat causally connected patch in an extended universe bounded by the Hubble horizon. (Our future domain of dependence.)
Probing accelerated expansion

\[
q \equiv -H^{-2} \frac{\ddot{a}}{a}
\]

\[
\dot{H} = \frac{\ddot{a}}{a} - H^2 = -H^2 (1 - q)
\]

\[
H = \left[ \frac{d}{dz} \left( \frac{d_L(z)}{1+z} \right) \right]^{-1}
\]

\[
H_0 d_L = z + \frac{1}{2} \left( 1 - q_0 \right) z^2 + \cdots
\]

\[
\mu(z) = m - M = 5 \log_{10} \left( \frac{d_L}{10 \text{ pc}} \right)
\]

Kinematic measurement of \( q(z) \) using \( d_L(z) \) only (independent of cosmological modeling, assuming standard candles)

Semiz, I., & Camlibel, K., 2015, arXiv:1505.04043v1

Frieman, Turner & Huterer 2008, ARAA 46 385
Accelerated expansion

Kirshner, R.P., 1999, PNAS, 96, 4224

Perlmutter et al. (1999), Riess et al. (1998)

$H_0 = 67.80 \pm 0.77, \quad \Omega_\Lambda = 0.692 \pm 0.010$

$q = \frac{1}{2} \Omega_M - \Omega_\Lambda < 0 \quad \text{(three-flat)}$
Observed deceleration parameter


\[ q = -H^2 \frac{\ddot{a}}{a} : \quad q(z) = q_0 + z \left( \frac{dq}{dz} \right)_0 + O(z^2) \]
Origin of accelerated expansion

The 3-flat FLRW line-element gives a viable geometry of post-inflation cosmology

A dynamical evolution of $a(t)$ is conceivably provided by general relativity, originally set-up as a four-covariant embedding of Newton’s law of gravity

Accelerated expansion requires the presence of negative pressure energy

Health warning:

the scale $H_0 c \sim 1 \text{ Angstrom/s}^2$ of cosmological gravitation is exceedingly weak, far below familiar Newtonian gravitational attraction
Observed scales of acceleration

galactic to cosmological scales

\[ a_0 \leq 1 \, \text{Å} \text{s}^{-2} \]

solar system

Quick fix?

Four-covariant stress-energy tensor (in null-space of Bianchi identity)

\[ \Lambda g^b_a = \Lambda \begin{pmatrix} 1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 \end{pmatrix}, \quad w = \frac{p_{DE}}{\rho_{DE}} = -1, \quad q = -1 \]

\[ T^{DE}_{ab} = -\Lambda g_{ab} \]
While the extant data are fully consistent with LambdaCDM, they do not exclude more exotic models of dark energy in which the dark energy density or its equation-of-state parameter vary with time.

Frieman, Turner & Huterer 2008, ARAA 46 385

Phenomenological descriptions

\[ w(a) = w_0 + w_a (1 - a) = w_0 + w_a \frac{z}{1 + z} \]

\[ q(z) = \frac{1}{2} \sum_i \Omega_i(z) \left[ 1 + 3w_i(z) \right] \]
New gravitational physics?

According to general relativity, cosmological evolution is open to any four-covariant feature.

The Hubble horizon is a four-covariant feature unique to cosmology (all inertial observers agree on the same Hubble radius).

Any back reaction thereof on a(t) will be four-covariant, i.e., is allowed.

\[ a_0 = \frac{cH_0}{2\pi} \approx 1 \text{ Å s}^{-2} \]
The Universe as a hologram?

General relativity:
thermodynamic limit of quantum spacetime

Information:
encoding microphysical distribution of matter on two-dimensional screens

Entropy from no-hair theorem:
classical limit hiding information of event horizons

Vacuum temperature:
surface gravity of Rindler horizon

‘t Hooft 1993 gr-qc/9310026
Suskind 1995 JMP 36 6377
Bekenstein, 1981, PRD 23, 287
Verlinde 2011 JHEP 4 29
van Putten 2012 PRD 85 064046
van Putten 2015 IJMPD 24 150024
Vacuum temperature: temperature of nearby H

- Black hole horizon
- Rindler horizon
- Mass infall (gravitational collapse)
- Observer at constant acceleration
Symmetric 2-sheet embedding of a black hole

van Putten 2010 CQG 27 075011

\[ ds^2 = -\tanh^2\left(\frac{\lambda}{2}\right)dt^2 + 4M^2 \cosh^4\left(\frac{\lambda}{2}\right)(d\rho^2 + \rho^2 d\theta^2 + \rho^2 \sin^2 \theta d\phi^2) \]

bifurcation horizon:

\[ A(\lambda) = 16M^2 \cosh^4\left(\frac{\lambda}{2}\right) \]
Black hole(s) in isotropic coordinates

\[ ds^2 = -N^2 dt^2 + \Phi^4 \left( d\rho^2 + \rho^2 d\theta^2 + \rho^2 \sin^2 \theta d\phi^2 \right) \]

\[ \Delta \Phi = 0 : \quad \Phi = 1 + \frac{M_1}{2|r - r_1|} + \frac{M_2}{2|r - r_2|} \]

Horizon area perturbations at turning points
Exact solution BH binding energy from Gibbs’ principle

Entropy from the “no hair” theorem (classically hiding maximal information)

\[ S_{AH,i} = \frac{1}{4} A_{H,i} f(\xi_1, \xi_2), \quad A_{H,i} = 16\pi M_i^2, \quad \xi_i = \frac{M_i}{a}, \quad f_i \equiv 1 + \xi_j \ (i \neq j) \]

Thermodynamic temperature

\[ T_{AH,i} = \left( \frac{\partial S_{AH,i}}{\partial M_i} \right)^{-1} = 4 \left( \frac{\partial A_{H,i}}{\partial M_i} f_i(\xi) + A_{H,i} \frac{\partial f_i(\xi)}{\partial M_i} \right)^{-1} \equiv \frac{4}{f_i(\xi)} \left( \frac{\partial A_{H,i}}{\partial M_i} \right)^{-1} \]

\[ dS_{AH,i} = \frac{1}{4} A_{H,i} df_i(\xi) \]

\[ T_{AH,i} dS_{AH,i} = A_{AH,i} \left( \frac{dA_{H,i}}{dM_i} \right) d\log f_i(\xi) \]

Gibbs virtual perturbations at the same total energy-at-infinity

\[ -dU = \left( T_{AH} dS_{AH} \right)_1 + \left( T_{AH} dS_{AH} \right)_2 \]

\[ U = -\frac{1}{2} M_1 \log f_1(\xi_2) - \frac{1}{2} M_2 \log f_2(\xi_1) \]
Binding energy from Gibbs’ principle

\[ x = \cos \theta, \quad s_i = \Phi^2 \left( \cos \lambda, \frac{\rho \sin \lambda}{\sqrt{1 - x^2}}, 0 \right) \tan \lambda(x) = -\sqrt{1 - x^2} f'(x), \quad \rho(x) = \rho_0 e^{f(x)} \]

\[ \lambda'(x) + 4 \rho \frac{\partial \rho}{\Phi} + 2 + 4 \tanh \lambda \frac{\partial \rho}{\Phi} + \frac{\tan \lambda}{\tan \theta} = 0, \quad \rho' = \rho \tan \lambda, \quad \frac{dA_H}{d\theta} = 2\pi \frac{\Phi^4 \rho^2}{\cos \lambda} \sin \theta \]
Newton’s law at turning points

At a binary separation $a$: 

$\xi_i = \frac{M_i}{a}$: \quad $A_{AH,i} = 16\pi M_i^2 f(\xi_j)$

$A'(\rho) = 0$: \quad $\rho = \frac{M_2}{2} \left( 1 - \frac{M_1}{2a} \right)$

$A_i(\rho) = 2\pi \int_0^\pi \Phi^4 \rho^2 \sin \theta \, dx = 4\pi \left[ 1 + \frac{M_2}{\rho} + \frac{M_1}{a} + \frac{M_2^2}{4\rho^2} + \frac{M_1 M_2}{2 \rho a} + \frac{M_2^2}{4 a^2} \right] \rho^2$

$A_{AH,2} = 16\pi M_2^2 f(\xi), \quad f(\xi) = 1 + \frac{M_1}{a} + \cdots$

Newton’s law

$S_{AH,2} = \frac{1}{4} A_{H,2} f(\xi), \quad A_{H,2} = 16\pi M_2^2, \quad f(\xi) = 1 + \frac{M_1}{a} + \cdots$
Signal horizon in cosmology

if they remain within a Hubble radius distance

\[ D^+ (\Sigma_t) \]

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Hubble flow through cosmological horizons

$$ds^2 = -dt^2 + a^2(t) \left( dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right)$$

de Sitter universe:

Hubble outflow

radiation dominated universe:

Hubble inflow

godesic observer

const spherical
surface area
Gibbons-Hawking temperature of de Sitter space

\[ ds^2 = -dt^2 + a^2(t) \left( dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right) \]

\[ H = \frac{\dot{a}}{a}, \quad R_H = H^{-1}: \quad ra = \sigma \left( 0 \leq \sigma < R_H \right) \]

\[
\begin{align*}
k_B T &= \frac{\hbar a_0}{2\pi c} = \frac{H_0 \hbar}{2\pi} \\
\end{align*}
\]

Gibbons-Hawking’s redshifted
Unruh temperature

\[ a_0 = a_{\text{loc}} \frac{d\tau}{dt} = \sigma H^2 : \quad \lim_{\sigma H \to 1} a_0 = H \]

\[ \Lambda = 8\pi \rho, \quad dE = \rho_\Lambda A_H dR, \quad S = \frac{1}{4} A_H : \quad T = \left( \frac{dS}{dE} \right)^{-1} \]

Stationary point of Helmholtz free energy
Surface gravity of Hubble horizon

\[ \sigma = ra : \]

\[ ds^2 = -dt^2 + a^2 dr^2 + \sigma^2 \left( d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right) = h_{ab} dx^a dx^b + \sigma^2 d\Omega^2 \]

\[ \begin{cases} \partial^c \sigma \partial_c \sigma = 0 \\ \kappa = \frac{1}{2} \nabla^2_{(2)} \sigma \end{cases} \]

Akbar & Cai 2007 PRD 75 084003

\[ a(t) = a_0 t^n, \quad \rho = rt^{\frac{n}{2}}, \quad R_H = H^{-1} = \frac{1}{n} t : \quad \kappa = (2n - 1)t^{n-1} \]

**Vanishing surface gravity in radiation dominated limit:**

\[ a = a_0 \sqrt{t}, \quad \kappa = 0 \quad \Rightarrow \quad T_H = 0 \]
Thermodynamic scales in gravitation

low temperature gravitation

\[ a_0 \leq 1 \text{As}^{-2} \]

high temperature gravitation

(independent of Hubble horizon)

Different approaches:

**Conventional:** evolution $a(t)$ governed by Einstein equations

**Holographic:** GR subject to thermodynamics of Hubble horizon

\[ L_0 = \frac{c^5}{G} \quad \rho_0 = -\frac{L_0}{cA_H} \]

\[ R_H = \frac{c}{H_0}, \quad A_H = 4\pi R_H^2, \quad \rho_c = \frac{3H_0^2}{8\pi G} \]

\[ \rho_\Lambda = -p_0 = \frac{L_0}{cA_H} = \frac{2}{3} \rho_c : \quad \Omega_\Lambda = \frac{2}{3} \]

Derives also from entropic forces (Easson et al. 2011, Phys. Lett. B 696 273)

Overall scale is remarkable for present Universe, but in contradiction with BBNS
Unruh (1976), Gibbons & Hawking (1977), Cai & Kim (2005)

\[ k_B T_H = \frac{\hbar a}{2\pi c}, \quad a = Hc\left(\frac{1-q}{2}\right): \quad k_B T = \frac{H\hbar}{2\pi}\left(\frac{1-q}{2}\right) \]

(The associated wave length < Hubble radius)

\[ \Omega_\Lambda = 2\left(\frac{1-q}{2}\right) = \left\{0, \frac{1}{2}, \frac{2}{3}\right\} \]

in radiation, matter and Lambda dominated epochs

\[ \Lambda = 8\pi\rho_\Lambda = (1-q)H^2 = H^2 + \frac{\ddot{a}}{a} \]
Mixed first and second order dark energy

\[ G_{ab} = 8\pi T_{ab} - \Lambda g_{ab} \]

\[ G_{ab} = 8\pi T_{ab} - H^2(1 - q)g_{ab} \]

Singular perturbation of the Hamiltonian energy constraint

\[ \tilde{G}_{ab} \equiv G_{ab} + \left( \frac{\ddot{a}}{a} \right) g_{ab} = 8\pi T_{ab} - H^2 g_{ab} \]

3-flat FRW line-element in 3+1:

\[ h_{ij} = a^2 \delta_{ij}, K_{ij} = -a \dot{a} \delta_{ij} : \quad (^3 R - K : K + K^2 = 16\pi \rho_m + 16\pi \rho_\Lambda \]

Now 2nd order in time:

\[ \frac{\ddot{a}}{a} = 2H^2 - 8\pi \rho_m : \quad c^{-1} \dot{R}_H = 1 + q = -1 + 3\Omega_m \]

Normalized:

\[ \tau = H_0 t : \quad a(t) \rightarrow a(\tau), \quad \omega_m = \frac{\rho_0}{\rho_c}, \quad h = \frac{H}{H_0} \]

\[ \frac{\ddot{a}}{a} = 2h^2 - 3\omega_m \left( \frac{a_0}{a} \right)^3 \]
Cosmological evolution of deceleration parameter

\[ z_t(q_0) = 0.43 - 0.24(1 + q_0) \]
Plane of \((q_0, (dq/dz)_0)\)

123-sigma contours of “Silver + gold” samples of Riess et al. 2004

\(\Lambda CDM\)

dynamical dark energy
Accelerated expansion $q_0<0$ is secure

Systematic uncertainties exist:

“gold” : Lambda CMD
“silver” : dynamical Lambda
“red” and “blue” Type Ia’s (Milne et al.2015)

Improved understanding will be crucial to differentiate dynamical and static Lambda

If dynamic, this will open a great new window to “Beyond Einstein Cosmology”
This late-time cosmology leaves BBNS unaffected \((q=1)\) with a cosmological density of DM:

\[
\Omega_m = \frac{1}{3} (2 + q) \cong 0.3 - 0.4
\]

Observed DM clustering on intermediate scales
(resolution \(\sim 10\) Mpc)

Vikram et al., 2015, astro-ph/1504.03002v2
Conclusions

Observed \((q_0, H_0)\) → observed values of \((z_t, (dq/dz)_0, a_0)\)

- Dynamical DE in a singular perturbation of GR
- Crucial is improved understanding of Type Ia data
- For late-time cosmology, BBNS unaffected \((q=1)\) with

\[
\Omega_m = \frac{1}{3} (2 + q) \cong 0.3 - 0.4
\]

- clustering of light CDM on super-galactic scales, not needed on galactic scales due to a modified Newton’s law:

van Putten (2014), arXiv:1411.2665v2

\[
\beta = \frac{T_0}{T} : \quad \text{high beta: Milgrom’s law} \quad \text{low beta: Newton’s law}
\]